

CORRELATION AND CRITICAL PATH ANALYSIS BASED ROAD TRAFFIC ROUTING ALGORITHM

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ABSTRACT

In this paper, we present a new dynamic road traffic routing algorithm, for enhanced clearing out of traffic from congested areas of the traffic network. The main objective of designing this algorithm is to implement a system that optimizes the rate of flow of traffic throughout the road network by minimizing traffic congestion rates. By cross-correlation analysis the system analyzes the relation between lanes, in particular, the rate of flow of traffic between lanes. It is used to compute the Time Difference of Arrival (TDOA), which in this context estimates the amount of time a fleet of traffic takes to travel from one intersection to the next. These time estimates are then used together with traffic counts at each intersection, and traffic weights which depict traffic flow patterns between lanes, to compute link scores and path scores for each road link and path, respectively. These scores are then presented as a network model based on the concerned road part of the road network. After all this, the algorithm uses this network model to compute an optimized sequential opening of the traffic lights in within that particular area. This algorithm is a dynamic model that mimics the operation of a police officer who controls traffic flow at road intersections during bad traffic congestion, the practice which is common here in Botswana and other third world countries.

KEYWORDS: *Road Traffic, Routing Algorithm, Ant Colony Optimization (ACO), Ant Dispersion Routing (ADR), Critical Path Analysis, Poisson Probability Distribution, Cross Correlation Analysis*

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I. INTRODUCTION

Road traffic congestion is proving to be a problem across the globe, and with motor vehicle numbers increasing significantly by the day. This phenomenon can only get worse if necessary measures are not taken. Traffic congestion can be caused by many factors ranging from vehicle breakdowns, bad roads, bad weather, faulty traffic control lights, high densities of vehicles on the roads and inefficient signaling methods, just to mention but a few. This problem has many implications in the economies and livelihoods of any country.

A lot of effort has been put in to curb this problem, with the development of new road traffic routing algorithms. Ant Colony Optimization (ACO) techniques have been used widely in the design of these algorithms; with Alves et al (2010) proposing a dynamic algorithm based on ACO called Ant Dispersion Routing (ADR) Algorithm [1].

Krömer et al (2011) presented a dynamic traffic routing approach that focused on the enhancement of routing in problematic areas of the road network like, accident scenes, road works and traffic jams, also based on ACO. The algorithm called Ant Colony Inspired Algorithm for Adaptive Traffic Routing uses a probability threshold to achieve sensitive and dynamic routing in a multi-path road network [2].

Kelly et al (2007) presented an inter – lane interactive algorithm called Decentralized Car Traffic Control using Message Propagation Optimized with a Genetic Algorithm [3] Shashikiran et al (2011) proposed a dynamic traffic routing protocol, for road traffic management during peak hours based on Kruskal's and Dijkstra's algorithms. In this protocol, Kruskal's algorithm is used to implement traffic routing and guidance part, while Dijkstra's algorithm is used to perform the optimal path discovery procedure based on the information availed by the vehicles' Dynamic Vehicles Navigation System (DVNS) [4].

BeeJam A Algorithm proposed by Wedde et al (2009) is a multi-agent bottom-up self - adaptive traffic routing protocol that is based on the behavior of the honey bee colony as they endure on their food searching endeavors [5].

In Botswana and in other developing countries police officers are tasked with the responsibility to control the traffic flow at the congested segments of the city road network during peak hours. This system has thus far proved to be working and highly efficient as compared to the currently used conventional system which uses pre-programmed traffic lights, which are unable to adapt to the variations in traffic density due to their fixed timing signaling.

We, therefore; here present a novel approach in traffic routing based on Critical Path Analysis (CPA) and Cross-Correlation Analysis (CCA). This traffic routing technique is a model that mimics the operation of the police officer system.

CPA is generally known as *network analysis* (NA). It can essentially be employed in any multi-task complex project to ensure that the full project is completed in the most minimum time and as efficiently as possible [6]. In this research, we seek to design a system that will ensure the maximum flow rate of traffic within a city road network. Cross-correlation analysis is a mathematical technique that can be used as a measure of the relation between any given time series [7]. It can also be used in the estimation of the Time Difference of Arrival (TDOA), which is widely used in applications like multi-channel synchronous data acquisition, real-time data processing and positioning [7, 8, 9].

The rest of the paper is organized as follows: The System Model is presented in Section II; Road Traffic Control Routing Algorithm Based on CPA is presented in Section III; Results and Analysis in Section IV and finally Conclusions in Section V.

II. SYSTEM MODEL

It is assumed that the traffic flowing from the source lanes follows a Poisson distribution, therefore the traffic flowing within all the other lanes follow the same pattern. The Poisson distribution is a discrete distribution probability mass function defined as follows [10];

$$.P(k/\lambda) = \frac{e^{-\lambda}\lambda^k}{k!} \quad (1)$$

In the road traffic framework, the variables in this equation represent the following parameters

- λ : the expected number of cars to arrive at the intersection in a given time period
- k : the number of cars actually arriving at the intersection at a particular instant
- $P(k/\lambda)$: the probability of cars arriving at an intersection, for an expected number of cars λ
- e : The exponential function.

Cross-Correlation Analysis

In this paper cross -correlation analysis is used to define how traffic flows on different lanes in the road network relate with one another. It helps in learning the lane to lane traffic dependencies and to compute the rate of traffic flow or delay between any two particular lanes. This comes as an improvement to the policeman approach, where there is no tool to precisely measure how the traffic flows at the nearby intersections relate with the each other.

The computation of cross-correlation is achieved by time-shifting one of the time series, then multiplying it with the other and summing up all the elements at each time shift. The value obtained from each of the above-mentioned computations is commonly referred to as the correlation coefficient. In mathematical terms the time shift value also known as the time-lag or delay, if not known is attained by finding the position of the highest correlation coefficient in the resultant series. If X and Y are two different time series of traffic counts on lanes x and y on the road network, their cross-correlation is given by the following equation;

$$.R_{xy}(\tau) = \frac{1}{N} \sum_{k=0}^{N-1} [x(k)][y(k-\tau)] \quad (2)$$

Where

τ – Time shift value, or the delay time is taken for the traffic to flow from one lane into another

k – Discrete time variable.

N – Number of samples/ timing intervals

Some time series can have associated dc – values which can mislead on the amount of relation between any given signals. To circumvent this, the mean of each series is subtracted from each of the element in the series. Taking λ_x and λ_y to be the mean values of the series $x(k)$ and $y(k)$, respectively the above formula changes to;

$$R_{xy}(\tau) = \frac{1}{N} \left(\sum_{k=0}^{N-1} [x(k) - \lambda_x][y(k-\tau) - \lambda_y] \right) \quad (3)$$

Statistical Analysis

This is done for the sole purpose of learning statistical dependencies between lanes and how they relate with one another. This is started off by computation of traffic means along each lane within the road network, over a predetermined number of timing intervals (i.e. 10 or 15 counts per lane). The means are then hierarchically ranked with respect to their

magnitudes and analyzed by the algorithm to figure out which lanes are the probable source lanes and to see which lane feeds into which lane. In this process, the lanes with higher traffic count means are taken to be probable source lanes, while those with lower means will be probably the outgoing or branching lanes.

After this analysis the traffic count fractions for each lane can be statistically estimated by the ratios of traffic count means for each branching lane to that of the corresponding source lanes. This is done for the sole purpose of picking the probabilistic traffic flow pattern from the source lanes into the branching lanes, for the computation of the routing algorithm. It can be simply defined as the use of past behavior of the system to predict its future behavior.

To illustrate this, figure 1 is used as a representation of a junction on the road network.

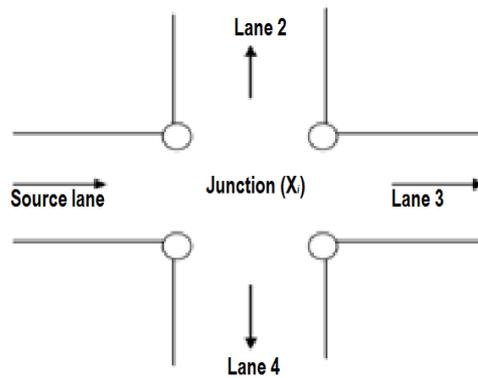


Figure 1: Example of Branching Lanes

If the lanes, lane 1 or the source lane, lane 2, lane 3 and lane 4 shown in figure 1, are respectively represented as; $s_j, x_1, x_2,$ and x_3 , the traffic flow weights can then be computed using equation (4). This equation is defined as the summation of fractions evaluated at individual timing instances, divided by the total number of timing intervals.

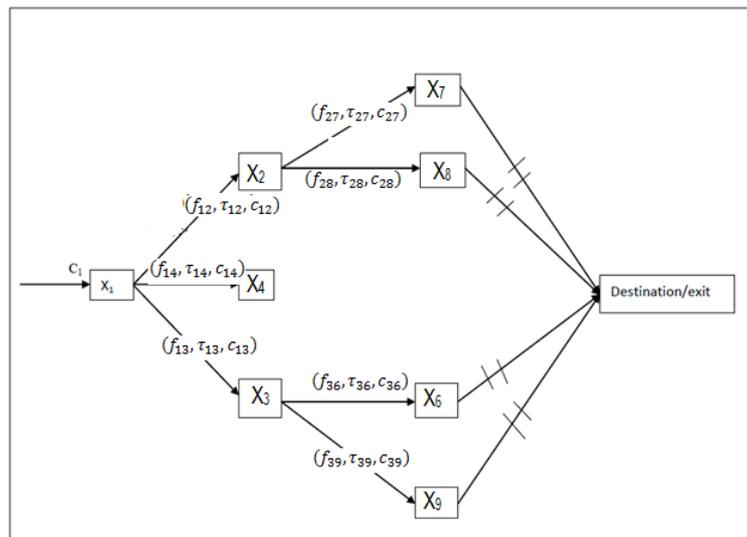


Figure 2: Empirical Road Network Model

$$f_j = \frac{1}{N} \sum_{i=1}^N \frac{x_j}{s_j} \tag{4}$$

Where

f_j – The estimated fraction of traffic flowing into the branching lane j

N – Number of timing intervals

n – Number of branching lanes

j - j^{th} Branching lane

i - i^{th} Timing instant

x_j – Traffic count on j^{th} branching lane

s_j – Traffic count on the j^{th} source lane.

It is worth noting that these fractions are estimated to be used for the computation of the routing algorithm, in the successive signaling instance. This is the other additional capability of this proposed system, using the past to predict the future. This is analogous to the policeman approach in which lane – to – lane traffic flow is carefully observed and eventually knowing which lane has a higher traffic flow, and subconsciously giving it a priority.

III. ROAD TRAFFIC CONTROL ROUTING ALGORITHM BASED ON CPA

The main purpose of this routing algorithm is to make decisions on the sequential opening of the road paths within the network, and it is achieved through critical path analysis optimization technique. This is where the routing routes decisions are taken solely based on the magnitudes of the complete path scores. The values of these path scores depend on the present traffic counts of the source lanes, time lags and the weights/ or the splitting ratio of traffic from the source lanes into the branching lanes. The larger path scores are given higher priority as compared to smaller path scores; therefore at every timing interval, the path scores are compared to pick the maximum in order to open its corresponding route. Figure 2 below shows the network of paths and links derived from the empirical road network model, which is used for simulations in this work.

It is very important to note that, the paths or links with shorter delays must be given priority when making routing decisions, as compared to those with longer delays. This is so because a shorter delay means that the traffic takes a shorter time to flow into that lane and therefore it needs more attention in order to avoid traffic build-ups. This may depend on several factors, like congestion or the distance between the two intersections. Therefore, a transformation that ensures higher time-lags are represented by smaller values as compared to smaller time-lags that will be represented by higher values is deduced. This will ensure that the routes with smaller delays have comparatively higher critical path scores.

To compute the time – lag scores for each lane, this transformation is devised by taking the reciprocal of each time – lag at each timing instance for individual traffic lanes. So, the following formula is used to deduce the scores corresponding to the relative time-lags.

$$\tau_{iscore} = \frac{1}{\tau_i}, i = 1, 2, \dots, n \quad (5)$$

Where

τ_{iscore} : Time – lag score for the i^{th} lane

τ_i : Time – lag for the i^{th} lane

From this the link score for the succeeding lane is deduced by-product of three parameters, being the statistical weight or fraction (f_i), the time – lag score (τ_{iscore}) and the traffic count of the current lane (c_{i-1}).

$$Linkscore (Li) = f_i \times \tau_{iscore} \times c_{i-1} \quad (6)$$

The path score of a particular route given by the summation of the link scores.

$$Path\ score\ (Pi) = \sum_{j=1}^N L_{ij}, j = 1, 2, \dots, n \quad (7)$$

The above equations simply define that, the link and the path score computations are dependent on the weight of the concerned lane with respect to its source lane (f_i), time – lag of the concerned lane with respect to its source lane (τ_i) and the traffic count of the source lane respectively (c_{i-1}).

Waiting Time

As already mentioned above, at each signaling instance, the routing decision is made by opening the route with the highest path score. Therefore, as a consequence depending on the traffic flow pattern and congestion on the individual lanes, some of the road links may take longer periods or even forever open, due to the continuous low traffic counts, hence low path scores. This will subsequently increase the waiting time of the traffic on those particular lanes, therefore causing a serious inconvenience to the road users.

To counter this problem, depending on the magnitudes of waiting times, the path scores for those links are multiplied by some integer factors. This will ensure that at the next signaling point those links take precedence, in the generation of the path opening sequence. These integer factors are chosen based on a number of factors like the length of waiting time, path scores and the importance of that particular link. This integer – factor multiplication is applied such that it takes the form of a staircase function, meaning that every unopened link at a signaling instance, its time-lag score is doubled at the next signaling point.

Weighting of Delayed Paths

The issue of queuing time is very crucial such that, if not taken into account can render the system dysfunctional, as some routes may take longer periods or even never open for traffic due to their continual low traffic counts and high delay values. To counter this effect, at every signaling point the time – lag scores for all the paths that were not opened in the last signaling point are multiplied by an integer factor. This process is termed *staircase multiplication* in this work. It amplifies the magnitudes of the time – lag scores of the non – opened lanes by the integer multiples at each timing instance. The link scores will continually increase taking the shape of a staircase as shown by figure 3, until a point where those path scores are highest, and the path can then be opened.

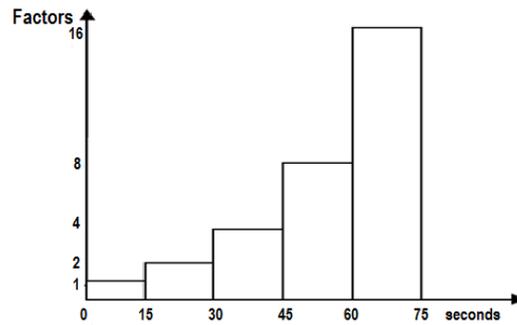


Figure 3: Staircase Factor Multiplications

In this algorithm, the routes with low traffic count have their path scores or time-lag scores doubled at each signaling instance until they have the maximum path score as compared to other routes, meaning they will be due to open for the traffic at the next signaling instant. After opening the factor multiplication start again from one and the process continues indefinitely, depending on the variations of traffic density on the road network.

The rate of recalculation is done based on the rate of change of traffic congestion inquest to try and cope with the unpredictable nature of traffic flow. At peak hours when the congestion is heaviest, it is very important to do recalculations at a slower rate, in order to clear as much as possible traffic queuing on the opened routes. Figure 4 below shows the traffic routing algorithm flow diagram.

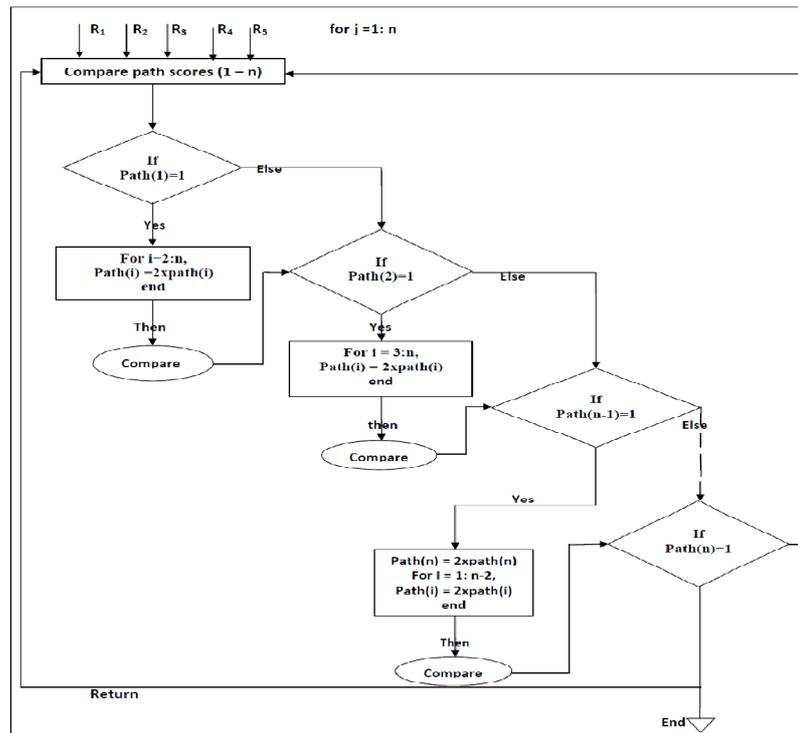


Figure 4: Traffic Routing Algorithm Diagram

IV.RESULTS AND ANALYSIS

Traffic Generation

The matrix below represents a distribution of traffic within the road network at different timing instances. The matrix is an average of 1500 matrices generated by the use of a random number generator MATLAB function called

Poisson. The random variables from this function follow Poisson distribution properties.

The elements of each row represent traffic counts for that particular lane at different timing instances, and elements of each column represent traffic counts on each lane at a particular timing instance. The zero values on rows 2 to row 8, show traffic delay times, they show that the amount of traffic that is supposed to flow from the source lanes into those particular lanes had not yet arrived or flowed into those lanes. They show after how many timing instances or delay, that the expected number of vehicles can smoothly move from the source lane to that particular lane. The timing instances are pre-determined timing intervals where traffic counts are recorded and they are represented by the columns in this matrix. Signaling and recording can, for example, be done at 10 seconds intervals, or even more depending on the traffic density.

If for example, three zeros appear before a non-zero value appears in one particular row, then the delay value is taken to be three or 45 seconds for traffic to smoothly flow from the source lane into that lane.

.X =

37 44 65 46 32 33 50 51 41 41

0 **14** 17 25 18 12 12 20 20 16

0 **4** 5 7 5 4 4 5 5 4

0 **19** 22 33 23 16 17 25 26 21

0 0 0 **4** 5 7 5 3 3 6

0 0 0 **10** 12 18 13 9 9 14

0 0 0 0 **7** 8 13 9 6 6

0 0 0 0 **12** 14 20 14 10 11

*Bolted figures indicate source lanes and their branching lanes.

By observing the above matrix, it is easy to figure out which lanes feed into which lanes. Visually looking at the matrix, and finding the delay value as mentioned above, it can be seen that lane 2, lane 3 and lane 4 are fed by lane 1 and they all have a delay of 1, by adding up the counts and counting the number of zeros before the first non-zero on each of the three lanes. The same process can be used to observe that lane 2 feeds lane 5 and lane 6 while lane 4 feeds lane 7 and lane 8, and the delay values are 3 for lane 5 and 6 and 4 for lane 7 and 8

Routing/ Optimization

The routing decisions are made based on the critical path analysis method, at every signaling point it compares the computed path scores and picks the highest in order to open its corresponding route next. As explained in chapter 3, the algorithm was designed in such a way that it can take into account several factors like; delays, traffic counts and the queuing time on each lane.

The constituents of the matrix R below are the code words for opening sequences of all the paths in the road network. The elements of each row, which are a string of 1's and 0's, represent the opening sequence for each path at different timing instances, and the elements of each column show which path is open for traffic at a particular instant in

time. An open path is represented by a 1 and the paths that are not open for traffic at that time are represented by a value of 0.

R =

0 0 1 0 0 0 1 0 0

0 1 0 0 0 1 0 0 1

0 0 0 0 1 0 0 0 0

0 0 0 1 0 0 0 0 0

1 0 0 0 0 0 0 1 0

Looking at the path opening sequence matrix above, it shows that at time $t = 0$, path 5 will have the highest path score and it will be opened, at $t = 1$, path 2 will be opened, path 1 will open at $t = 3$, path 4 will open at $t = 4$ and so forth.

Simulations

For comparison purposes, three traffic control systems were simulated. The proposed system together with the police officer system and the fixed – timing pre-programmed conventional system were simulated using the same input parameters, to compute average travel time in each system. The simulations were carried out at different traffic densities to measure the performance and capability of the systems under these conditions. These simulation conditions are shown in table 1 below and Table 2 shows another of the parameters which are essential in these simulations, which signal timings for the fixed timing conventional system.

Table 1: Simulation Conditions for the Three Traffic Control Systems

Traffic Density	Traffic Count	Average Speed (km/h)	Delay Period (s)
Normal traffic	$\lambda \leq 50$	60	20
Moderate congestion	$50 < \lambda \leq 200$	50	25
Peak hour	$\lambda > 200$	40	30

The average speed of 60 km/h, was chosen for these simulations because of its standard maximum speed for urban traffic in Botswana and many other countries throughout the world. The other speeds of 50 km/h and 40 km/h for moderate and peak hour traffic were logically picked, looking at the fact that the mobility is expected to decrease as numbers increase on the road network.

Table 2: Traffic Signal Timings for the Conventional System

Traffic Signal	Signal Timing (s)
Green	10
Red	10
Amber	3

The computations of travel times were done using the main source lane, X_1 in figure 2 as the reference starting point for all simulations. The computations comprised of summing up all the delay values for each route, also taking into account the traffic signal timings for the conventional system, and then taking the average time of travel for each system. The police officer system was modeled by modifying the waiting time parameter of the proposed system. A staircase multiplication of 1.5 was used for the police officer system instead of 2 of the proposed system.

The average speed parameter was used to estimate the rate of traffic dissolution by counting the number of cars let to pass at each opening, and how many are left behind on the source lane for the conventional system. For the proposed system and the police officer systems, simulations are done under the assumption that, all the traffic is cleared from the source lane at each opening.

$$\text{Leaving traffic} = \frac{\text{averagespeed} \times 1000 \times \text{signal timing}}{3600 \times \text{average car length}} \tag{8}$$

$$\text{Remaining traffic} = n - \frac{\text{averagespeed} \times 1000 \times \text{signal timing}}{3600 \times \text{average car length}} \tag{9}$$

Figures 5, 6 and 7 below are graphs showing average travel times for each system. Each system was simulated using parameters in table 1.

Figure 5: Travel time comparison at $\lambda = 45$

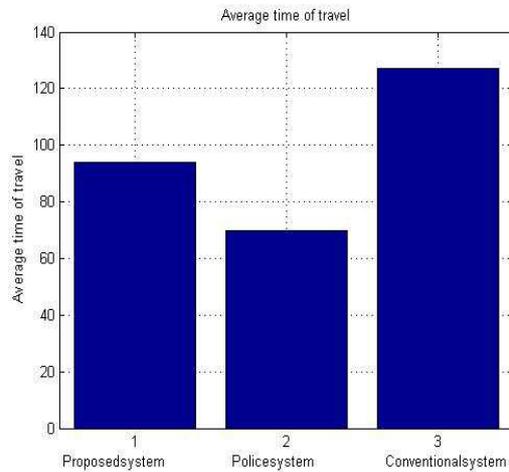


Figure 6: Travel Time Comparison at $\lambda = 80$

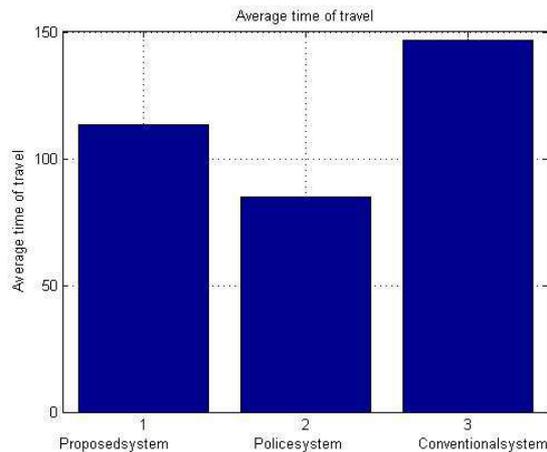


Figure 7: Travel Time Comparison at $\lambda = 210$

Figure 8 below shows a plot of the travel times against the number of vehicles queuing for all the three systems. The plot shows that the police system outperforms the proposed and the conventional system. This is so because of the obvious reason that it is being controlled practically by a human being, because realistically no machine can beat human intelligence.

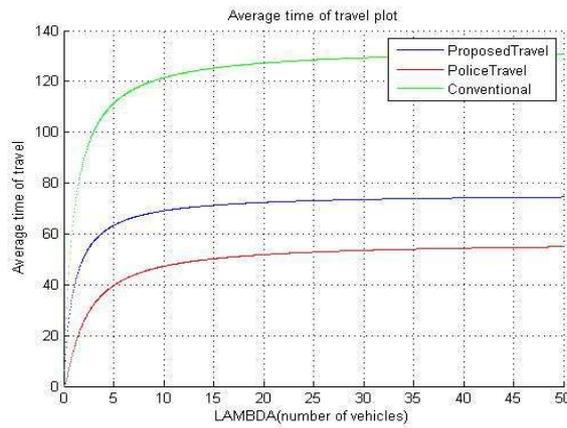


Figure 8: Plot Showing Travel Time of the Systems against the Number of Vehicles Queuing

Table 3, 4 and 5 below summarize the results from the graphs presented above. They also show the percentage improvements on the police officer system and the proposed systems as compared to the conventional system.

Table 3: Travel Time Comparison at $\lambda = 45$ (Normal Traffic)

System	Average Travel Time (s)	%Improvement	Leaving Traffic
Proposed system	74.35	30.6 %	cleared
Police system	54.69	49.0%	cleared
Conventional system	107.20	N/A	34.72

Table 4: Travel Time Comparison at $\lambda = 80$ (Moderate Traffic)

System	Average Travel Time (s)	%Improvement	Leaving Traffic
Proposed system	93.83	25.9 %	cleared
Police system	69.68	45.7%	cleared
Conventional system	127.01	N/A	30.56

Table 5: Travel time comparison at $\lambda=210$ (Peak Traffic)

System	Average Travel time (s)	%Improvement	Leaving traffic
Proposed system	113.4597	23.1 %	cleared
Police system	84.9038	42.1%	cleared
Conventional system	146.9005	N/A	29.86

V. CONCLUSIONS

This paper has presented a novel idea in road traffic routing, whose aim is to achieve a near seamless traffic flow within the road network, that can also cope with the variations in traffic densities more especially during the peak hours. This novel idea is a routing algorithm based on Critical Path Analysis (CPA). Also discussed in this paper are some of the existing traffic routing algorithms and some ideas from the past researches.

This novel idea, as compared to other previous researches and the already existing traffic control systems referenced in this paper, it is better and more effective looking at its simple yet very efficient operation and its logistical deployment. The road network is divided into smaller networks, each with a processing unit responsible that section of the network, as compared to the Message Propagation system in which every intersection has its own dedicated processing unit. This is an advantage for this new model as one processing unit takes care of a larger part of the road network. The routing decisions are taken based on the traffic density, delay and the traffic splitting rate throughout all the routes, and the route opening sequence is generated with the aim of giving priority to the routes with higher traffic counts and shorter

delays, but at the same time catering for the other routes with lower traffic counts and longer delays, hence reducing network deficiency.

Through its periodical traffic count updates from the traffic sensor network, the system has a capability of building its own Self Organising Traffic Map (SOTM), which is able to change with respect to the variations of traffic densities within the network.

The system was simulated against the conventional system that is currently used in Botswana and other parts of the world, together with the police system. The simulations show great improvement by this proposed system on the travel times of traffic within the road network. The results show that there is a great improvement in travel times for both the proposed system and police system, with the police system having more percentage improvement from the conventional system, as compared to the proposed system. This is so because the police system in this work is taken as the ideal system that is being mimicked by the proposed system. However, the advantages of the proposed system outweigh this disparity in performance between the two systems. The proposed system is better than the police officer system because it does not require any labor to operate as compared to the police system which is very labor intensive.

One pain that is evidently observed here is the declining numbers of traffic leaving the source lane intersection for the conventional system, as the queue length increases. This is due to the reduction in mobility of traffic as the congestion increases.

The above results show that there is a great improvement in travel times for both the proposed system and police system, with the police system having more percentage improvement from the conventional system, as compared to the proposed system. This is so because the police system in this work is taken as the ideal system that is being mimicked by the proposed system. One pain that is evidently observed here is the declining numbers of traffic leaving the source lane intersection for the conventional system, as the queue length increases. This is due to the reduction in mobility of traffic as the congestion increases.

REFERENCES

1. D. Alves, J. van Ast, Z. Cong, B De Schutter, and R. Babuska, "Ant Colony optimization for traffic dispersion routing", *Proceedings of the 13th International IEEE Conference on Intelligent Transportation Systems (ITSC 2010)*, Madeira Island, Portugal, pp 683 – 688, Sept. 2010.
2. P. Krömer, J. Martinovič, M. Radecký, R. Tomis, and V. Snášel, "Ant Colony Inspired Algorithm for Adaptive Traffic Routing", *Department of Computer Science, VŠB – Technical University of Ostrava, 17. Listopadu 15, 708 33, Ostrava – Poruba, Czech Republic, 978-1-4577-1124-4/11 ©2011 IEEE*.
3. M. Kelly, and G. D. MarzoSerugendo, "Decentralized Car Traffic Control using Message Propagation Optimized with a Genetic Algorithm", *School of Computer Science and Information Systems, Birkbeck College, London, 1-4244-1340-0/07 ©2007 IEEE*.
4. V. Shashikiran, T. T. S. Kumar, N. S. Kumar, V. Venkateswaran and S. Balaji, "Dynamic Road Traffic Management based on Krushkal's Algorithm", *IEEE – International Conference on Recent Trends in Information Technology, ICRTIT 2011, MIT Anna University, Chennai, June 3 – 5, 2011, 978 – 1 – 4577 – 0590 – 8/11 ©2011 IEEE*.

5. H. F. Wedde, S. Lehnhoff, S. Senge and A. M. Lazarescu, "Bee Inspired Bottom-Up Self-Organization in Vehicular Traffic Management", School of Computer Science, Technical University of Dortmund, 2009 Third IEEE International Conference on Self-Adaptive and Self-Organizing Systems, 978-0-7695-3794-8/09, ©2009 IEEE, DOI 10.1109/SASO.2009.24, IEEE Computer Society.
6. E. Crisostomi, S. Kirkland, and R. Shorten, "Robust and risk-averse routing algorithms in road networks", Preprints of the 18th IFAC World Congress, Milano (Italy) August 28 – September 2, 2011, Copyright by the International Federation of Automation Control (IFAC).
7. H. C. Yan and W. Liu, "Design of Time – Difference of Arrival Estimation System Based on Fast Cross Correlation", National Key Laboratory of Electronic Measurement Technology, North University of China, Taiguan, China, 978-1-4244-5824-0 ©2010 IEEE Volume 2, no. 464
8. F. Gustaffson and F. Gunnarson, "Positioning Using Time - Difference of Arrival Measurements", Department of Electrical Engineering, Linköping University, SE – 58183 Linköping, Sweden, 0-7803-7663-3/03 ©2003 IEEE, ICASSP 2003.
9. M. Azaria and D. Hertz, "Time Delay Estimation by Generalized Cross Correlation Methods", IEEE Transactions on Acoustics, Speech, and Signal Processing, Volume ASSP – 32, No 2, April 1984, 0096 – 3518/84/0400 – 0280, ©1984 IEEE
10. J. C. B. Cooper, "The Poisson and Exponential Distributions", Published by the Applied Probability Trust, ©Applied Probability Trust 2005.
11. S. Raychaudhuri, S. J. Mason, R. R. Hill, L. Monch, O. Rose, T. Jefferson, J. W. Fowler eds., "Introduction to Monte Carlo Simulation", Oracle Crystal Ball Global Business Unit, 390 Interlocked Crescent, Suite 130, Broomfield, C. O. 8002, U.S.A., Proceedings of the 2008 Winter Simulation Conference, 978-1-4244-2708-6/08 ©2008 IEEE.
12. V. T. N. Nha, S Djabel, and J. M. Lero, "A Comparative Study of Vehicles' Routing Algorithms for Route Planning in Smart Cities", UCD School of Computer Science and Informatics, Ireland.
13. S. Messelodi, C. M. Modena, M. Zanin, F. G. B. De Natale, F. Granelli, E. Betterle and A. Guarise, "Intelligent Extended Floating car Data collection, Expert systems with Applications", volume 36, pp 4213 – 4227, 2009, ELSEVIER, Science Direct.
14. Bhupendra Kumar Soni et al., Performance Analysis of XY Routing Algorithm for 2d Mesh Topology Noc Architecture Under Different Traffic Pattern, TJPRC:International Journal of Power Systems & Microelectronics (TJPRC:IJPSM), Volume 2, Issue 2, November-December 2017, pp. 35-42
15. B. S. Kerner, C. Demir, R. G. Herrtwich, S. L. Klenov, H. Reborn, A. Hang and A. G. Daimler Chrysler, "Traffic State Detection with Floating Car Data in Road Networks", Proceedings of the 8th International IEEE Conference on Intelligent Transportation Systems, Vienna Austria September, 13 – 16 2005, 0 – 7803 – 9215 – 9/05 © 2005 IEEE.
16. T. Jeske, Institute for Security and Distributed Applications, Hamburg University of Technology, "Floating Car Data from Smart phones: what Google and Waze know about you and how Hackers can control traffic.

17. *D. Holliday, "Critical Path Method", March 18, 2009, Compute Science Department, California State University, Fullerton.*
18. *J. W. Chinneck, "Binary and Mixed Integer Programming, Practical Optimization: A Gentle Introduction, Chapter 8", 2010, <http://www.sce.carleton.ca/faculty/chinneck/po.html>*